CURRENT DISTRIBUTION IN SUPERCONDUCTING STRIP TRANSMISSION LINES*

D. M. Sheen, S. M. Ali, D. E. Oates†, R. S. Withers†, and J. A. Kong

Department of Electrical Engineering and Computer Science
Research Laboratory of Electronics
Massachusetts Institute of Technology
Cambridge, MA 02139

†Lincoln Laboratory, Massachusetts Institute of Technology
Lexington, MA 02173

ABSTRACT

A method for the calculation of the current distribution, resistance, and inductance for superconducting strip transmission lines is presented. These calculations allow accurate characterization of both high-T_c and low-T_c superconducting strip transmission lines. For a stripline geometry the current distribution, resistance, and inductance are calculated as a function of the penetration depth for various film thicknesses. These calculations are then used to determine the penetration depth for a $YBa_2Cu_3O_{7-X}$ superconducting thin film from the measured temperature dependence of the resonant frequency of a stripline resonator. The power dependence of the $YBa_2Cu_3O_{7-X}$ surface resistance is shown plotted against the RF magnetic field which is determined from the calculated current distribution.

I. INTRODUCTION

The calculation of the electrical parameters of superconducting transmission lines is important both for design of circuitry and for characterization of the superconductors. Microwave frequency measurements using stripline or microstrip resonators can determine the fundamental physical properties of these films, such as penetration depth and intrinsic surface resistance. To accurately determine these properties from the measurements requires accurate calculation of the current distribution, resistance, and inductance of the stripline or microstrip.

To date, researchers have relied on very approximate means of relating the quality factor or Q of the resonator to the intrinsic surface resistance of the film[1–3]. One method often used is the incremental inductance method first suggested by Wheeler[4]. This method assumes that the penetration depth is small compared to the thickness of the superconductor. This assumption is often invalid for the new high-T_c superconductors which may have penetration depths comparable to typical film thicknesses.

Previous research into the calculation of the transmission line properties of superconductors has often been limited to simple geometries. However, work on practical transmission line structures has also been done. Sass and Stewart[5] obtained the self and mutual inductances of a superconducting transmission line system using an assumed current distribution. Alsop et al.[6] used a numerical solution based on the finite difference method. More recently, Chang[7] used a variational technique to calculate the inductances of a multi-superconductor transmission line system.

In this paper, the method and results for the calculation of the current distribution, resistance, and inductance, as functions of the penetration depth for a superconducting strip transmission line are presented. Unlike the previous work, resistance will be considered in the formulation as well as inductance. The method used is a modification of the method used by Weeks et al. for the analysis of normally conducting transmission line systems[8]. The inductance calculation is then used to determine the penetration depth at zero temperature $\lambda(0)$ for a $YBa_2Cu_3O_{7-X}$ superconducting thin film. The calculations are also used to determine the intrinsic surface resistance as a function of RF magnetic field for the $YBa_2Cu_3O_{7-X}$ resonator to show nonlinear effects.

II. PROBLEM FORMULATION

Consider M coupled superconducting transmission signal lines and a return path of one or more lines, all of rectangular cross section in a uniform, lossless dielectric with permittivity ϵ and permeability μ_0, as shown in Figure 1. The constitutive relation between the current density in the superconducting lines and the electric field is modeled using the two-fluid model, in which the total current is the sum of both normal current and supercurrent. The normal current is assumed to obey Ohm's law and the supercurrent is assumed to obey the London equation. The two components of the current may be combined and related to the electric field by defining a complex conductivity σ_c,

$$\overline{J} = \sigma_c \overline{E} = (\sigma_1 - j\sigma_2)\overline{E} = \left(\sigma_1 - \frac{j}{\omega\mu_0\lambda^2}\right)\overline{E} \qquad (1)$$

where λ is the penetration depth of the superconductor. If the TEM mode is assumed and the complex conductivity is used, the problem becomes analogous to the normal conductor skin effect problem and may be solved using a modification of the method described in [8].

To enable numerical solution of the problem, the conductors are subdivided into a total of $N + 1$ small rectan-

*This work was conducted under the auspices of the the Consortium for Superconducting Electronics with full support by the Defense Advanced Research Projects Agency contract MDA 972-90-C-0021.

OF-I

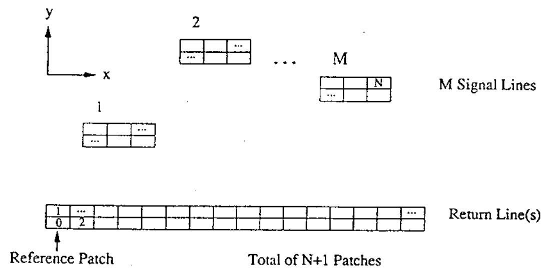

Figure 1. Multiple transmission line system configuration.

gular patches numbered $0, 1, 2, \ldots, N$ as shown in Figure 1. Patch number 0 is defined to be the reference patch. If $\bar{v}$ is defined to be the vector of voltages on the patches relative to the reference patch, and $\bar{i}$ is the vector of currents flowing in the patches, the system can be considered to be a system of N coupled transmission lines which will obey the transmission line equation,

$$\frac{d\bar{v}}{dz} = -(\bar{\bar{r}} + j\omega\bar{\bar{l}}) \cdot \bar{i} \tag{2}$$

where $\bar{\bar{r}}$ is the matrix of self and mutual resistances per unit length between the patches, and $\bar{\bar{l}}$ is the matrix of self and mutual inductances per unit length between the patches. To proceed with the solution, two fundamental requirements are made; the total current in the system is forced to equal zero, and the current density in each patch is assumed to be uniform.

Since the conductivity of each patch is complex, a complex resistance is defined by $r_n = 1/(\sigma_c)_n A_n$, where A_n is the area of the n^{th} patch and $(\sigma_c)_n$ is the complex conductivity of the n^{th} patch. The resistance matrix elements $r_{m,n}$ are obtained from the real part of the complex resistance of the reference patch plus that of the m^{th} patch if $m = n$,

$$r_{m,n} = Re\left(\frac{1}{(\sigma_c)_0 A_0} + \delta_{m,n}\frac{1}{(\sigma_c)_m A_m}\right) \tag{3}$$

where $\delta_{m,n} = 1$ for $m = n$ and $\delta_{m,n} = 0$ for $m \neq n$.

The elements of the inductance matrix are slightly more complicated. The kinetic contribution to the inductance is obtained from the imaginary part of the complex resistance,

$$l^{(k)}_{m,n} = \frac{1}{\omega} Im\left(\frac{1}{(\sigma_c)_0 A_0} + \delta_{m,n}\frac{1}{(\sigma_c)_n A_n}\right) \tag{4}$$

A partial inductance is defined [8]

$$l^{(p)}_{m,n} = \frac{-\mu_0}{4\pi A_m A_n} \iint_{S_m} \iint_{S_n} ln\left[(x-x')^2 + (y-y')^2\right] dx'dy'dxdy \tag{5}$$

where S_m is the cross section of the m^{th} patch and S_n is the cross section of the n^{th} patch. From stored energy concepts, it may be shown that the m, n^{th} element of the inductance matrix is given by

$$l_{m,n} = l^{(k)}_{m,n} + l^{(p)}_{m,n} - l^{(p)}_{m,0} - l^{(p)}_{0,n} + l^{(p)}_{0,0} \tag{6}$$

The integrals in (5) may be done in closed form [7]. Practical experience using the analytic formula for the partial inductance integration has shown that for small patches that are far apart the evaluation of this integral fails due to finite computer precision (double precision). Where the exact formula fails, a numerical procedure (Gaussian Quadratures) may be used to evaluate the integrals.

When the impedance matrix $\bar{\bar{z}}$ is formed and the voltages on the lines are set (usually 1 for patches on the signal line and 0 for patches on the return line(s)), the currents on the patches may be immediately solved for by inverting the impedance matrix to obtain the admittance matrix $\bar{\bar{y}}$,

$$\bar{i} = -\bar{\bar{z}}^{-1} \cdot \frac{d\bar{v}}{dz} = -\bar{\bar{y}} \cdot \frac{d\bar{v}}{dz} \tag{7}$$

To determine the resistance per unit length matrix $\bar{\bar{R}}$ and inductance per unit length matrix $\bar{\bar{L}}$ for the original system of M coupled transmission lines with $\bar{V}$ the vector of voltages on the signal lines and $\bar{I}$ the vector of currents on the signal lines, the transmission line equation

$$\frac{d\bar{V}}{dz} = -(\bar{\bar{R}} + j\omega\bar{\bar{L}}) \cdot \bar{I} = \bar{\bar{Z}} \cdot \bar{I} = \bar{\bar{Y}}^{-1} \cdot \bar{I} \tag{8}$$

is used. The elements of admittance matrix $\bar{\bar{Y}}$ are obtained from admittance matrix $\bar{\bar{y}}$ by summing the admittances that are in parallel,

$$Y_{ik} = \sum_{m=N_i^{(i)}}^{N_i^{(f)}} \sum_{m=N_k^{(i)}}^{N_k^{(f)}} y_{m,n} \tag{9}$$

where $N_i^{(i)}$ and $N_i^{(f)}$ are the first and last patch numbers for the i^{th} signal line and $N_k^{(i)}$ and $N_k^{(f)}$ are the first and last patch numbers for the k^{th} signal line. After $\bar{\bar{Y}}$ has been calculated it may be inverted to obtain the impedance matrix $\bar{\bar{Z}}$. The resistance and inductance per unit length matrices are then obtained from the real and imaginary parts of $\bar{\bar{Z}}$. For the stripline configuration there is only one signal line and therefore $\bar{\bar{R}}$ and $\bar{\bar{L}}$ reduce to scalars.

III. NUMERICAL RESULTS

In this section, numerical results for the stripline resonator configuration shown in Figure 2 will be presented. These results include the current distribution, inductance, and resistance as functions of the penetration depth for various film thicknesses.

Current distributions for the stripline were calculated using (7) and are shown in Figure 2 for a penetration depth of 0.2μm with a film thickness $t = 0.3\mu$m.

The inductance was calculated using (8) and is shown in Figure 3 for film thicknesses from 0.1μm to 0.8μm. As expected, the inductance increases with penetration depth. The increase is due to two factors; the internal inductance rises as the current density becomes more uniform, and the

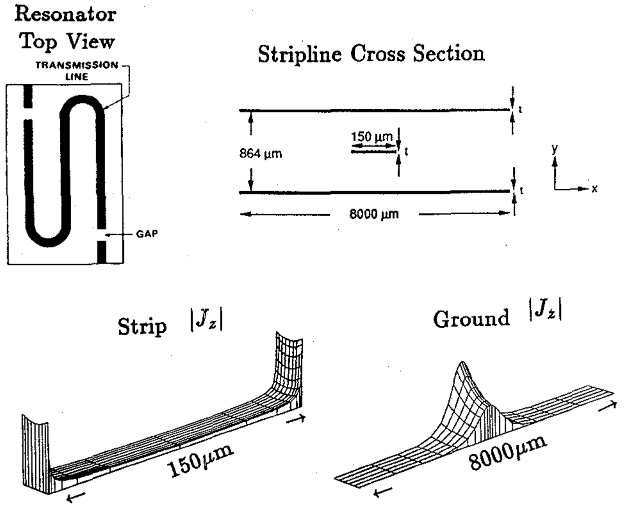

Figure 2. Stripline resonator configuration and current density distributions for film thickness $t = 0.3\mu$m and $\lambda = 0.2\mu$m.

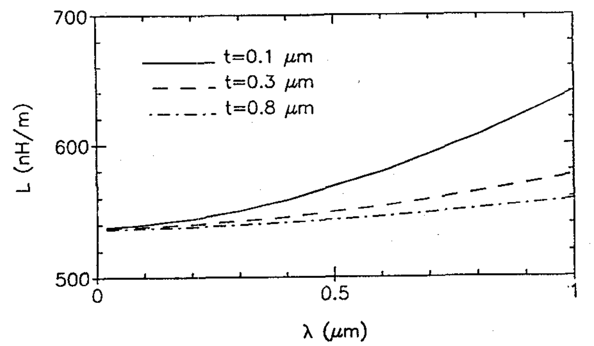

Figure 3. Inductance vs. penetration depth λ for various film thicknesses t.

kinetic inductance rises as the penetration depth increases.

The resistance was calculated using (8) and is shown in Figure 4 for a frequency of 1 GHz and the real part of the conductivity equal to 10^6 (1/Ωm). The resistance for other values of frequency and conductivity may be easily determined because the current distribution is essentially independent of frequency and the real part of the conductivity. The resistance is proportional to the real part of the conductivity times the frequency squared, $R \propto \sigma_1 f^2$. For small penetration depths the resistance is very low and is essentially independent of the thickness of the film, as expected because the current is crowded to the edges and corners of the superconductors, thus increasing the thickness will have little effect on the area in which the current is flowing. At larger penetration depths, the resistance is essentially inversely proportional to the film thickness, as it would be for a uniform current distribution.

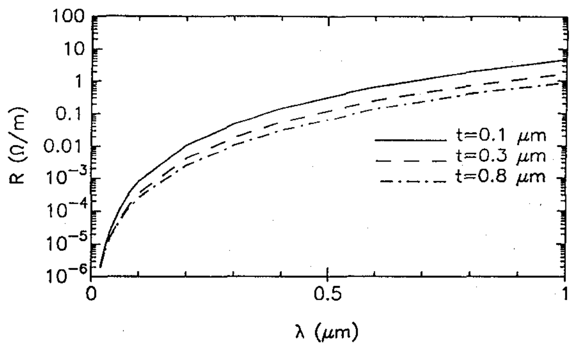

Figure 4. Resistance vs. penetration depth λ for various film thicknesses t.

IV. RESONATOR MEASUREMENTS

To accurately characterize the superconducting thin films, and as an experimental verification of the calculations, the inductance per unit length is used to obtain the magnetic penetration depth at zero temperature $\lambda(0)$ from measurements of the resonant frequency of a stripline resonator as a function of temperature for a sputtered thin film of superconducting $YBa_2Cu_3O_{7-X}$. The stripline resonator is described in [1]. The change in resonant frequency is very small, thus highly accurate calculation of the inductance is required. The resonant frequency of the stripline resonator is inversely proportional to the square root of the inductance per unit length of the line. The inductance is a function of the penetration depth which, in turn, is a function of the temperature. The penetration depth as a function of temperature $\lambda(T)$ is assumed to be given by the two-fluid model $\lambda(T) = \lambda(0)/(1-(T/T_c)^4)^{-1/2}$. A fitting procedure is then applied to the calculated and measured resonant frequency curves to determine the penetration depth $\lambda(0)$. Figure 5 shows the measurements using a film of the high-T_c superconductor $YBa_2Cu_3O_{7-X}$ with a film thickness of 0.3 μm. The best fit is obtained with $\lambda(0) = 0.167$ μm. Similar results for $\lambda(0)$ have been obtained by others[9].

The calculated resistance and inductance values may now be used to extract the intrinsic surface resistance R_s of the thin film superconductors from the measured Q of the stripline resonator. Using the penetration depth, obtained as discussed above, and the measured Q, the resistance per unit length of the stripline is determined. The calculated values of resistance then allow extraction of the real part of the conductivity and therefore the intrinsic surface resistance R_s, which for $\sigma_2 \gg \sigma_1$ is defined as $R_s = (1/2)\omega^2\mu_0^2\sigma_1\lambda^3$.

As an example of the quantitative information which one can extract from the stripline measurements when the current distribution is known, Figure 6 shows the dependence of R_s on the peak value of the RF magnetic field, H_{rf}, for $YBa_2Cu_3O_{7-X}$ films sputtered at three different temperatures. To obtain these results, R_s was measured as a function of input power to the resonator and H_{rf} was

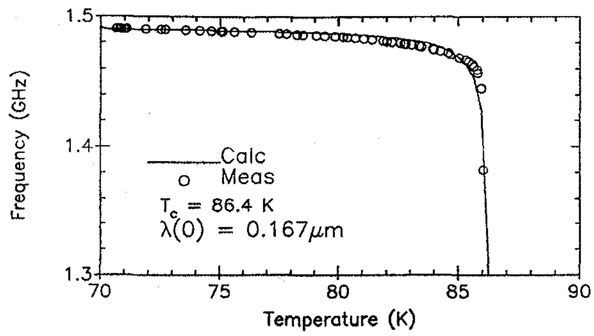

Figure 5. Resonant frequency vs. temperature. Determination of the penetration depth of $YBa_2Cu_3O_{7-X}$ measured and calculated values. Film thickness $t = 0.3$ μm.

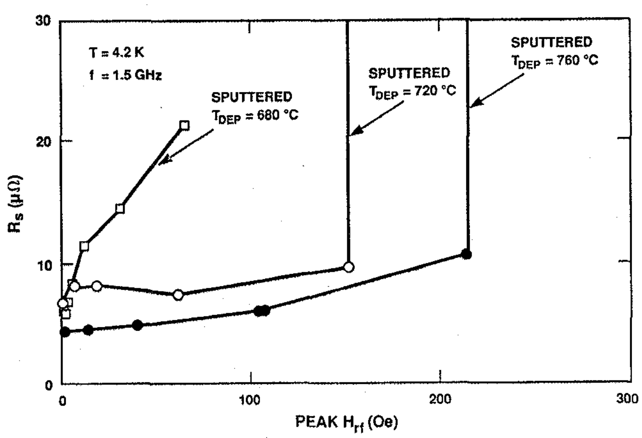

Figure 6. Surface resistance vs. peak magnetic field for several sputtered thin films of $YBa_2Cu_3O_{7-X}$.

computed from the total RF current in the stripline, from λ, and from the calculated current distribution. The films sputtered at higher temperatures clearly show a greatly reduced dependence of R_s on current and H_{rf}. For all of these films the input power was increased until a sharp increase in R_s was measured, indicating that the critical current or magnetic field was reached. The best film set shows R_s with little dependence on H_{rf} up to 225 Oe which is to our knowledge the largest reported value of H_{rf} for which R_s remains unchanged.

V. CONCLUSIONS

A method for the calculation of current distribution, resistance, and inductance for superconducting transmission line systems has been demonstrated by presenting results using a stripline configuration. Accurate calculation of the inductance is of critical importance for design of superconducting transmission lines and for characterizing the superconducting materials. The effectiveness of the inductance calculations has been demonstrated in the determination of the penetration depth $\lambda(0)$ from measurement of the temperature dependence of the resonant frequency of a $YBa_2Cu_3O_{7-X}$ stripline resonator. The resistance values calculated are useful both for the design of superconducting transmission lines and for the conversion of measurements of resonator Q to the intrinsic surface resistance of the material. The current distribution may be used to calculate kinetic inductance and the RF magnetic field. Using these calculations the power dependences of the surface resistance of several sputtered $YBa_2Cu_3O_{7-X}$ thin films were plotted as a function of peak RF magnetic field. To obtain accurate results for the new high-T_c films it is necessary that the resistance calculations take into account the fact that the film thickness may be comparable to λ, as in our calculations.

ACKNOWLEDGEMENT

The authors would like to thank Michael Tsuk for providing a computer code for normal conductors that implements the method described in Weeks et al.

REFERENCES

[1] D. E. Oates and A. C. Anderson, "Stripline measurements of surface resistance: relation to HTSC film properties and deposition methods," SPIE, Vol. 1187, pp. 326–337, 1989.

[2] A. D. MacDonald, S. A. Long, J. T. Williams and D. R. Jackson, "Microwave and millimeter wave characterization of high temperature superconducting thin films," 1990 IEEE AP-S Int. Symp. Proc., pp. 712–715, 1990.

[3] M. S. DiIorio, A. C. Anderson, and B. Y. Tsaur, "RF surface resistance of Y-Ba-Cu-O thin films," *Phys. Rev. B*, Vol. 38, No. 10, pp. 7019–7022, 1988.

[4] H. A. Wheeler, "Formulas for skin effect," *Proc. IRE*, Vol. 30, pp. 412–424, 1942.

[5] A. R. Sass and W. C. Stewart, "Self and mutual inductances of superconducting structures," *J. Appl. Phys.*, Vol. 39, No. 4, pp. 1956-1963, 1968.

[6] L. E. Alsop, A. S. Goodman, F. G. Gustavson, and W. L. Miranker, "A numerical solution of a model for a superconductor field problem," *J. Comp. Phys.*, No. 31, pp. 216–239. 1979.

[7] W. H. Chang, "Numerical calculation of the inductances of a multi-superconductor transmission line system," *IEEE Trans. Magn.*,Vol. MAG-17, No. 1, pp. 764–766, Jan. 1981.

[8] W. T. Weeks, L. L. Wu, M. F. McAllister, and A. Singh, "Resistive and inductive skin effect in rectangular conductors," *IBM J. Res. Develop.*, Vol. 23, No. 6, pp. 652–660, Nov. 1979.

[9] S. M. Anlage, B. W. Langley, H. J. Snortland, C. B. Eom, T. H. Geballe and M. R. Beasley, "Magnetic penetration depth measurements with the microstrip resonator technique," . *J. Superconductivity*, Vol. 3, No. 3, 1990.